On The Moebius and Euler Totient Functions Calculation

Gevorg Hmayakyan

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Abstract

In this paper the $a_{n,m} = a_{n-1,m} - a_{n-1,m-n+1}$ recursively defined sequence arithmetical properties are investigated. Using this sequence the Moebius and Euler Totient functions are calculated.

1 Introduction

The $a_{n,m}$ is defined as a z coefficients of $\prod_{i=1}^{n-1}(z^i - 1)$ product expansion:

$$\prod_{i=1}^{n-1}(z^i - 1) = \sum_{i=1}^{n(n-1)/2} a_{n,i} z^i.$$

From the

$$\prod_{i=1}^{n-1}(z^i - 1) = (z^{n-1} - 1)\prod_{i=1}^{n-2}(z^i - 1)$$

follows:

$$a_{n,m} = a_{n-1,m} - a_{n-1,m-n+1}.$$

Let define the $b_{n,t}$ as:

$$b_{n,t} = \sum_{k=0}^{N_{n,t}} a_{n,kn+t} \tag{1}$$

where

$$N_{n,t} = \left[\frac{n-1}{2} - \frac{t}{n}\right]$$

In this terms:

$$b_{n,q} = c_q(n)$$

where $c_q(n)$ is a Ramanujan's sum. From this obviously follows that:

$$b_{n,1} = \mu(n)$$

and

$$b_{n,0} = \phi(n),$$

where $\mu(n)$ and $\phi(n)$ are Moebius and Euler Totient functions accordingly. The C.A.Nicol result [1] is used in order to prove the main identity.

2 Main Result

Theorem 1:

$$b_{n,q} = c_q(n)$$

where $b_{n,q}$ is defined above at 1 and $c_q(n)$ is a Ramanujan's sum.

Proof: It is well known that the $c_q(n)$ equals to von Sterneck's arithmetic function [2]:

$$\Phi_{n,k} = \frac{\phi(n)\mu\left(\frac{n}{\gcd(n,k)}\right)}{\phi\left(\frac{n}{\gcd(n,k)}\right)}.$$

From the other hand this equals to $b_{n,q}$ according to (15a) of [1]. So we have:

$$c_q(n) = \Phi_{n,q} = b_{n,q}$$

which proves the theorem.

3 Bibliography

References

[1] C.A.Nicol *On Restricted Partitions and a Generalization of the Euler ϕ Number and the Moebius Function.* 1953.

[2] B. Berndt *commentary to On certain trigonometrical sums..., Ramanujan, Papers, p. 371*